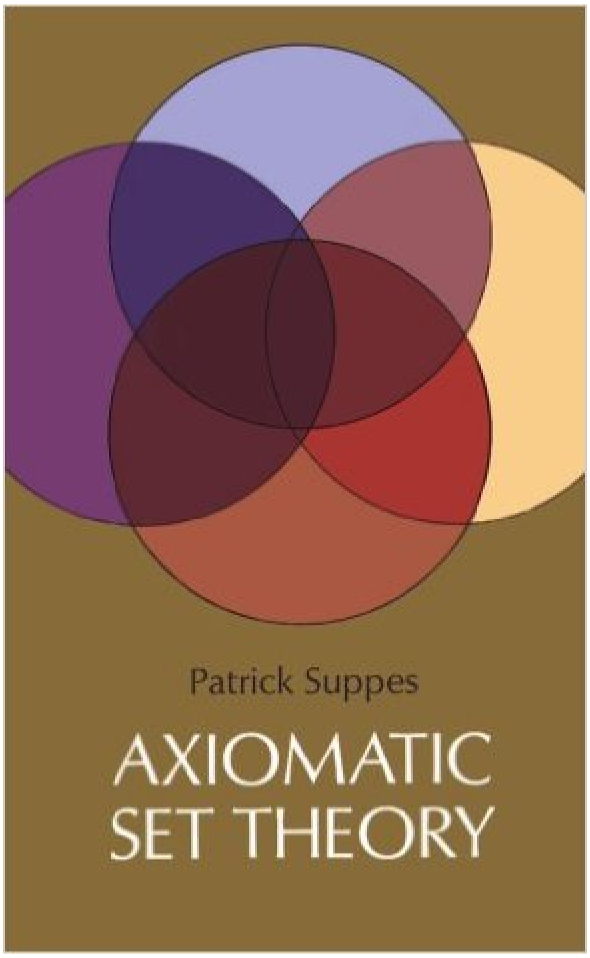
Axiomatic Set Theory (Dover Books On Mathematics)



Synopsis

One of the most pressingproblems of mathematics over the last hundred years has been the question: What is a number? One of the most impressive answers has been the axiomatic development of set theory. The question raised is: "Exactly what assumptions, beyond those of elementary logic, are required as a basis for modern mathematics?" Answering this question by means of the Zermelo-Fraenkel system, Professor Suppes' coverage is the best treatment of axiomatic set theory for the mathematics student on the upper undergraduate or graduate level. The opening chapter covers the basic paradoxes and the history of set theory and provides a motivation for the study. The second and third chapters cover the basic definitions and axioms and the theory of relations and functions. Beginning with the fourth chapter, equipollence, finite sets and cardinal numbers are dealt with. Chapter five continues the development with finite ordinals and denumerable sets. Chapter six, on rational numbers and real numbers, has been arranged so that it can be omitted without loss of continuity. In chapter seven, transfinite induction and ordinal arithmetic are introduced and the system of axioms is revised. The final chapter deals with the axiom of choice. Throughout, emphasis is on axioms and theorems; proofs are informal. Exercises supplement the text. Much coverage is given to intuitive ideas as well as to comparative development of other systems of set theory. Although a degree of mathematical sophistication is necessary, especially for the final two chapters, no previous work in mathematical logic or set theory is required. For the student of mathematics, set theory is necessary for the proper understanding of the foundations of mathematics. Professor Suppes in Axiomatic Set Theory provides a very clear and well-developed approach. For those with more than a classroom interest in set theory, the historical references and the coverage of the rationale behind the axioms will provide a strong background to the major developments in the field. 1960 edition.

Book Information

Series: Dover Books on Mathematics

Paperback: 288 pages

Publisher: Dover Publications; 1st edition (June 1, 1972)

Language: English

ISBN-10: 0486616304

ISBN-13: 978-0486616308

Product Dimensions: 5.4 x 0.6 x 8.4 inches

Shipping Weight: 10.4 ounces (View shipping rates and policies)

Average Customer Review: 3.8 out of 5 starsÂ Â See all reviewsÂ (16 customer reviews)
Best Sellers Rank: #637,888 in Books (See Top 100 in Books) #74 inÂ Books > Science & Math > Mathematics > Pure Mathematics > Set Theory #321 inÂ Books > Science & Math > Mathematics > Pure Mathematics > Logic #6016 inÂ Books > Textbooks > Science & Mathematics > Mathematics

Customer Reviews

Mathematics is a first order theory whose primitive formulaeall take the form ’a is a member of b’. ’a’ can be a set or atom; ’b’ must be a set. If you do not object to the preceding sentence, then read on.Axiomatic Set Theory (AST) lays down the axioms of the now-canonical set theory due to Zermelo, Fraenkel (and Skolem), called ZFC. Building on ZFC, Suppes then derives the theory of cardinal and ordinal numbers, the integers, rationals, and reals, and the transfinite--Cantor’s paradise. Suppes accomplishes in 250 well laid out pages what required 800 crabbed pages in Principia Mathematica.This book evolved out of a class Suppes taught at Stanford in the long ago 1950s. It has since remained the best book of its kind. The reason is that subsequent presentations of set theory are too difficult, too contrived, too clever by half. They disdain the basics as old hat.AST has several valuable pedagogical features.1. The introduction to relations and functions is the best I know of. I am disappointed at how little attention has been devoted to relations and relational algebra in recent decades.2. Suppes has a nice way of introducing a simple axiom, then showing that that axiom is a theorem when a more complicated axiom is later introduced. In particular, he develops the theory of cardinals by means of a temporary axiom to the effect that equipollent sets have identical cardinalities. This axiom becomes a theorem when the axiom of Choice is introduced in the final chapter.The axiom schema of Replacement is introduced as late as possible, to enable transfinite arithmetic. He then turns around and shows that Replacement makes Subsets and Pairing redundant.

One does not hear about set theory too much these days, no doubt due to the de-emphasis of foundational discussions in mathematics. Foundational questions of course were the focus of much attention in mathematics in the early twentieth century, this taking place because of the many paradoxes in set theory and due to the influence of the philosophers. Set theory, the theory of types, and mathematical logic are still very important though in computer science and in artificial intelligence, due to the needs in these fields for knowledge representation, computational models of intelligence, and automated reasoning. This book could serve to introduce these topics or as an historical reference to the issues as they were hotly debated in the last century. The first chapter gives an informal introduction to the notion of a set, first-order predicate logic (notions of bound and free variables and quantification), and the Zermelo-Fraenkel axioms of set theory. The author describes the difficulties in the "axiom of abstraction" in the writings of Frege as pointed out by Bertrand Russell. It is pointed out that the axiom of abstraction is in fact an infinite collection of axioms, thus motivating the concept of an "axiom schema". The axiom schema that is used explicitly in the book is the "axiom schema of separation" due to Ernst Zermelo, which he formulated in order to make precise the notion of a statement as being "definite". More of the set-theoretic paradoxes are discussed, along with their classification due to F.P. Ramsey into "linguistic" and "semantical" ones. The advantage of an older book on set theory is that more of the underlying details are explained, instead of just being formally developed.